COMMUNICATIONS TO THE EDITOR

Dear Editor:

This is in response to the communication by Meter and Bird (*1*) and their comments concerning the origin of a certain Reynolds number expression for annuli in which a particular geometric parameter is included in the Reynolds number to correct for the curvature of the surfaces. These authors are of the opinion that this particular Reynolds number group [their Equation (8)] was first proposed by Fredrickson and Bird (*2*). We wish to point out that this group actually appears much earlier in the literature. For example Wiegand and Baker (*3*) write

$$N_{Re} = \frac{D(1-a)V\rho}{\mu}\left[\frac{1}{\phi(a)}\right] \quad (1)$$

where

$$\frac{1}{\phi(a)} = \frac{(1+a^2) + (1-a^2)/\ln a}{(1-a)^2} \quad (2)$$

Wiegand and Baker, as well as others, cite Lamb (*4*) as the source of Equation (1); however the expressions to be found in reference *4* have to do with the velocity distribution and volume flux in annuli and not the specific form of Equation (1).

Fredrickson and Bird (*2*) write

$$N_{Re}(n,a) = \frac{D^n V^{2-n}\rho/m}{2^{n-3}\dfrac{(1-a^2)^{n+1}}{(1+a)}}\left[\Omega_p\left(\frac{1}{n},a\right)\right]^n \quad (3)$$

The Reynolds number expression Meter and Bird (*1*) have reference to is that of the Newtonian case, that is ($n = 1$), of Equation (3):

$$N_{Re} = \frac{DV\rho}{\mu}\left[\frac{4\Omega_p(1,a)}{(1-a^2)^2}\right] \quad (4)$$

Now in another place (*5*) Fredrickson and Bird write

$$\Omega_p(1,a) = \frac{Q}{\pi D^4 J/32\mu} \quad (5)$$

but it can be readily shown that

$$\Omega_p(1,a) = \frac{(1+a)(1-a)^3}{4\phi(a)} \quad (6)$$

When one substitutes Equation (6) into Equation (4) and rearranges, there results

$$N_{Re} = \frac{DV\rho}{\mu}\left[\frac{1-a}{\phi(a)}\right]$$

which is simply a restatement of Equation (1). This Reynolds number group has been cited by numerous authors, including Carpenter and associates (*6*),* Tao and Donovan (*7*),* and ourselves (*8*).

In this connection certain variations of Equation (1) for Newtonian flow in annuli are worth noting. Walker and associates (*9*) write

$$N^{+}_{Re} = \frac{D(1-a)V\rho}{\mu}\left[\frac{1+a^2-2\lambda^2}{(1-a)(1-\lambda^2)}\right] \quad (7)$$

Introducing

$$\lambda^2 = \frac{1-a^2}{2\ln 1/a} \quad (8)$$

into Equation (7) one gets

$$N^{+}_{Re} = \frac{D(1-a)V\rho}{\mu}\left[\frac{(1-a)}{\phi(a)(1-\lambda^2)}\right] \quad (9)$$

Wilcox (*10*) writes

$$f = \frac{c(n,a)}{N_{Re}^{*}(n,a)} \quad (10)$$

for the Power law flow model. Here for $n = 1$ Equation (10) reduces to

$$N_{Re}^{+} f = 16 \quad (11)$$

In contrast Lorentz and Kurata (*11*) write

$$N_{Re}^{++} = \frac{de\,V\rho}{\mu} \quad (12)$$

where

$$de = D\sqrt{(1+a^2)-(1-a^2)/\ln 1/a} \quad (13)$$

Expressing Equations (12) and (13) in terms of the parameter (a) one gets

$$N_{Re}^{++} = \frac{D(1-a)V\rho}{\mu}\left[\frac{1}{\sqrt{\phi(a)}}\right] \quad (14)$$

NOTATION

- a = ratio of radius of inner cylinder to that of outer cylinder
- $c(n,a)$ = function defined by Equation (6) in reference *10*
- de = equivalent diameter, Equation (13)
- D = diameter of outer cylinder
- f = friction factor
- J = pressure gradient across annulus
- $m,n,$ = parameters in power law flow model
- N_{Re} = Reynolds number, Equation (1)
- $N_{Re}(n,a)$ = Reynolds number, Equation (3)
- N_{Re}^{+} = Reynolds number, Equation (7)
- N_{Re}^{*} = Reynolds number, defined by Equation (3) in reference *10*
- N_{Re}^{++} = Reynolds number, Equation (12)
- Q = volumetric rate of flow
- V = average axial fluid velocity

Greek Letters

- λ = dimensionless radial distance of maximum velocity in annulus, Equation (8)
- μ = viscosity
- π = 3.1416
- ρ = density
- ϕ = function of radii ratio, Equation (2)
- $\Omega_p\left(\frac{1}{n},a\right)$ = function given in Table 3 of reference *5*
- $\Omega_p(1,a)$ = function defined by Equations (5) or (6)

* The form of Equation (1) presented by these authors contain certain purely typographical errors.

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Yours very truly,

J. G. Savins
Socony Mobil Oil Company, Incorporated, Dallas, Texas